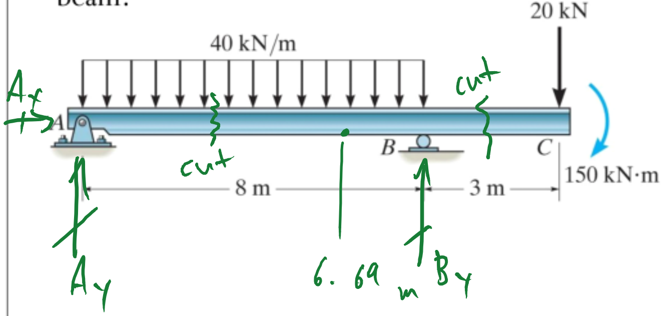
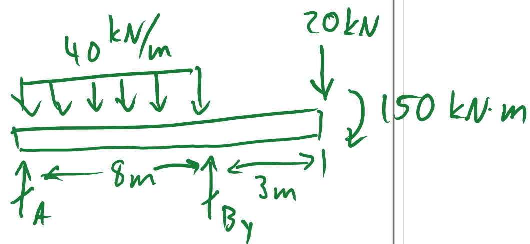
Obtain the expressions for V(x) and M(x) and draw the shear and bending moment diagram for the beam.



Discontinuities in V(x) & M(x) occur at point loads and couples respectively. "Cut" the beam between the discontinuities.

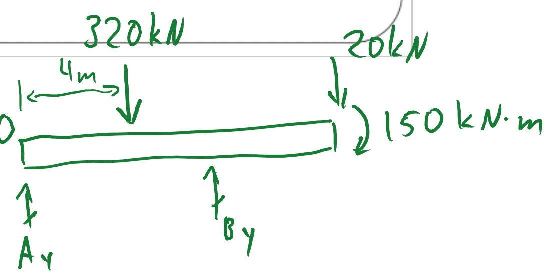
Cut between A & B and between B & C

Solve for A_x, A_y, B_y (reactions)



$\Sigma F_x = 0 \Rightarrow A_x = 0$

$(\Sigma M)_A = 0$



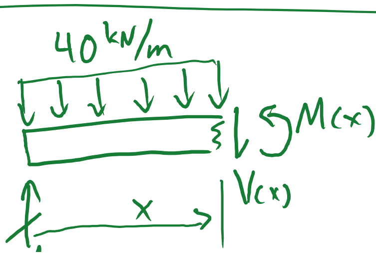
$\Rightarrow -(4m)(320kN) + (8m)B_y - (11m)(20kN) - (150kNm) = 0$

$\therefore \boxed{B_y = 206.25 \text{ kN}}$

$\Sigma F_y = 0 \Rightarrow A_y + B_y - 320kN - 20kN = 0$

$A_y + B_y = 340kN \Rightarrow \boxed{A_y = 133.75 kN}$

Cut between A & B $(0 < x < 8m)$



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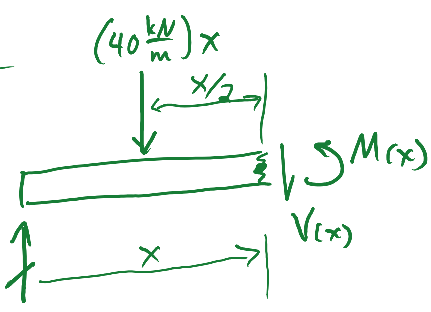
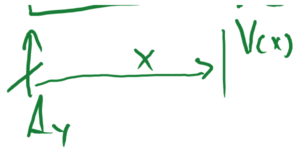
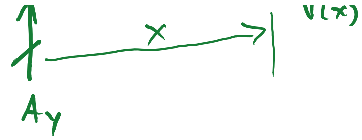






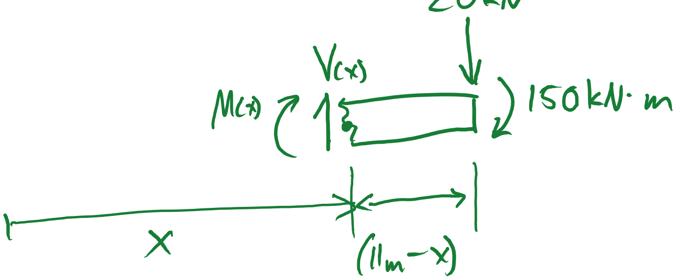
$$\Sigma F_y = 0 \Rightarrow A_y - \left(40\tfrac{kN}{m}\right)x - V(x) = 0$$

$$\boxed{V(x) = (133.75\,kN) - \left(40\tfrac{kN}{m}\right)x \quad ; \quad 0 < x < 8m}$$

$$(\Sigma M)_x = 0 \Rightarrow M(x) + \underbrace{\left(40\tfrac{kN}{m}\right)\cdot x}_{\text{force}} \cdot \underbrace{\frac{x}{2}}_{\text{arm}} - \underset{\text{force}}{A_y} \cdot \underset{\text{arm}}{x} = 0$$

$$\boxed{M(x) = -\left(20\tfrac{kN}{m}\right)x^2 + (133.75\,kN)\cdot x = 0 \quad ; \quad 0 < x < 8m}$$

Cut between B & C $(8m < x < 11m)$



$$\Sigma F_y = 0$$

$$\Rightarrow V(x) - 20kN = 0$$

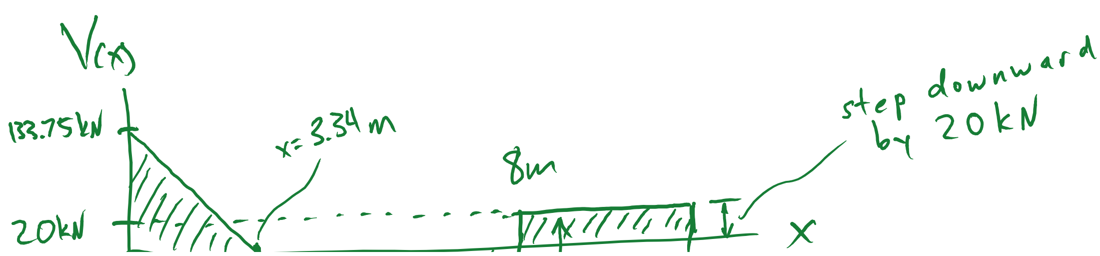
$$\boxed{V(x) = 20kN \quad ; \quad 8m < x < 11m}$$

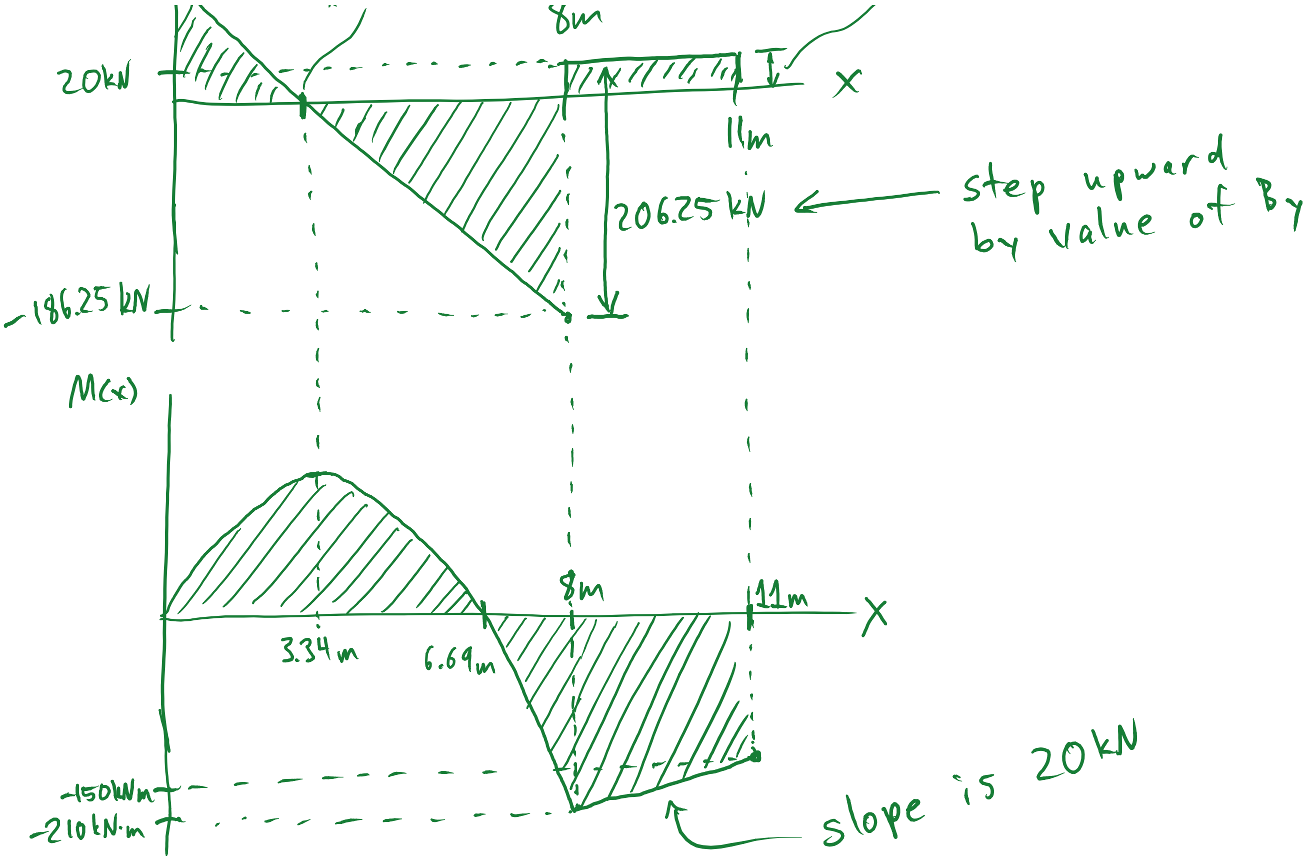
$$(\Sigma M)_x = 0$$

$$\Rightarrow -M(x) - (150\,kN\cdot m) - \underbrace{(11m - x)}_{\text{arm}}\underbrace{(20kN)}_{\text{force}} = 0$$

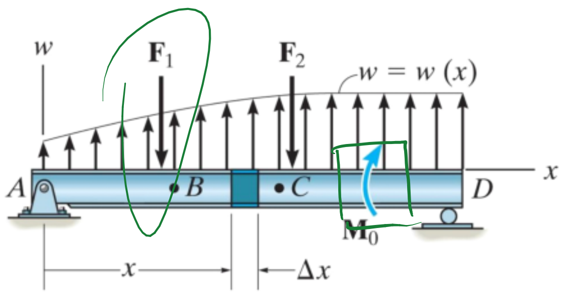
$$M(x) = -150kN\cdot m - 220kN\cdot m + (20kN\cdot m)\cdot x$$

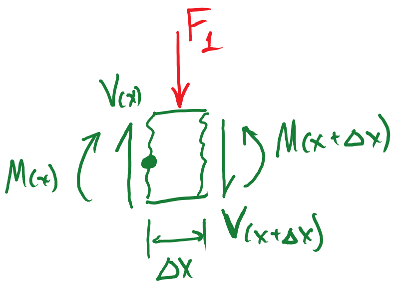
$$\boxed{M(x) = (20kN)x - 370\,kN\cdot m \quad ; \quad 8m < x < 11m}$$




8m
20kN
X
11m
206.25 kN
step upward by value of By
-186.25 kN
M(x)
8m
11m
X
3.34m
6.69m
-150kNm
-210kN·m
slope is 20kN

Relations Among Load, Shear and Bending Moments





$\Sigma F_y = 0 \Rightarrow V(x) - V(x+\Delta x) - F_1 = 0$

$V(x+\Delta x) - V(x) = -F_1$

take $\Delta x \to 0$

$\Rightarrow$ step in $V(x) = |F_1|$ down

$(\Sigma M)_x = 0$

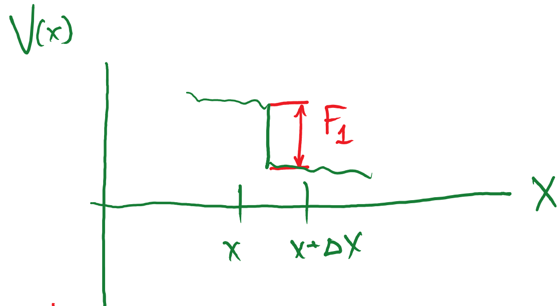
$\Rightarrow -M(x) + M(x+\Delta x) - F_1 \cdot \frac{\Delta x}{2} - V(x+\Delta x)\cdot \Delta x = 0$

$M(x+\Delta x) - M(x) = F_1 \cdot \cancel{\frac{\Delta x}{2}}^{\,0} - V(x+\Delta x)\,\cancel{\Delta x}^{\,0} = 0$

take $\Delta x \to 0$

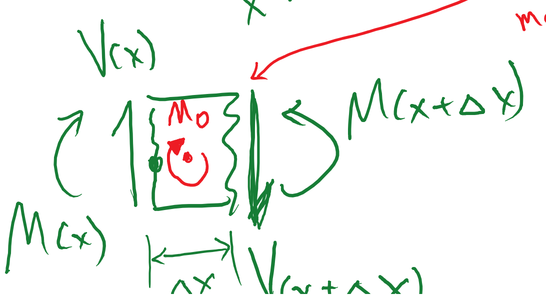
$\Rightarrow M(x+\Delta x) - M(x) = 0$

$\boxed{\text{No step!}}$



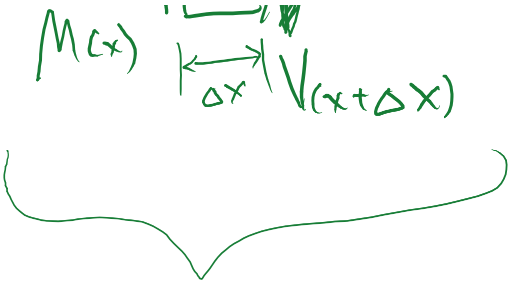
M_0 is between x and $(x+\Delta x)$

the couple has magnitude M_0, but is in $-\hat{k}$ direction



$(\Sigma M)_x = 0$

$-M(x) + M(x+\Delta x) - M_0 - \Delta x \cdot V(x+\Delta x) = 0$



$$-M(x) + M(x+\Delta x) - M_0 - \Delta x \cdot V(x+\Delta x)$$

$$\underbrace{M(x+\Delta x) - M(x)}_{\text{Value of step}} = M_0 + \underbrace{\cancel{\Delta x}}_{0} \cdot V(x+\Delta x)$$

take $\Delta x \to 0$

$$\Rightarrow \boxed{M(x+\Delta x) - M(x) = M_0}$$

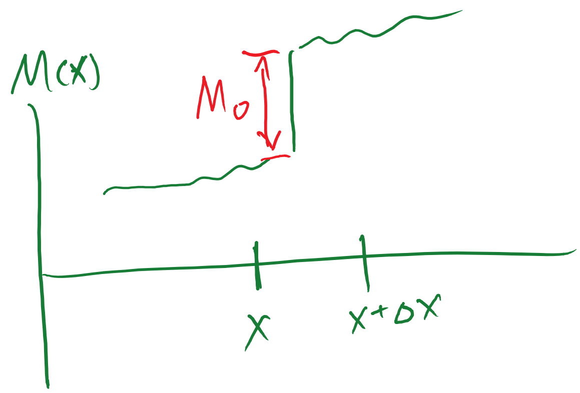
step is upward!

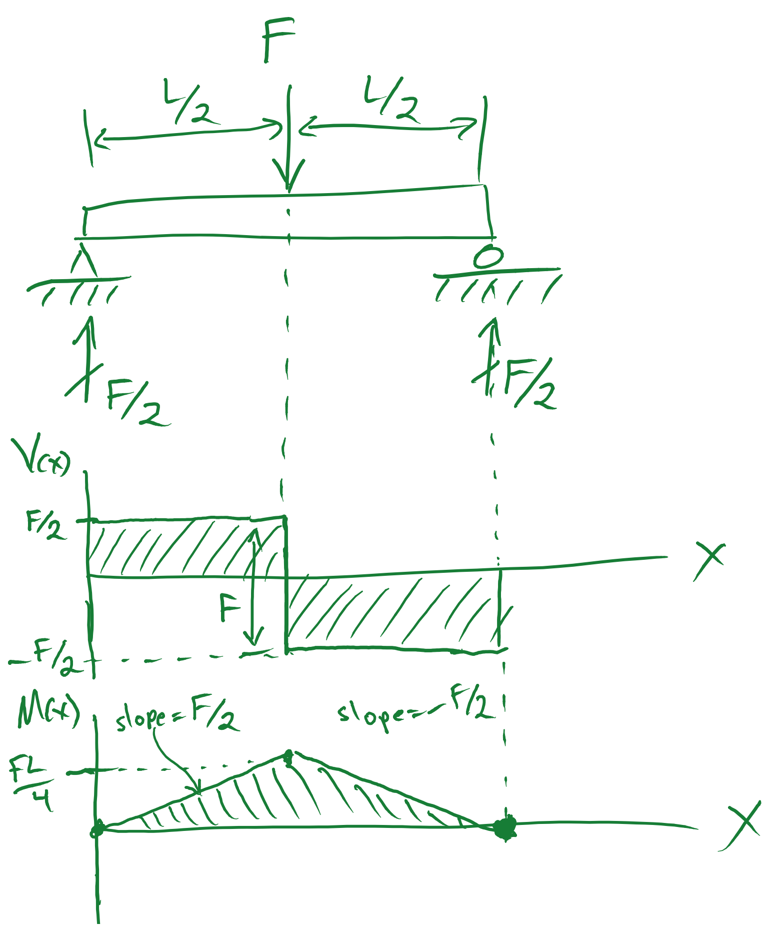
$$\Sigma F_y = 0$$

$$V(x) - V(x+\Delta x) = 0$$

$$\Rightarrow V(x+\Delta x) = V(x)$$

$\Rightarrow$ $\boxed{\text{No step in } V(x)}$





$$\boxed{\frac{dM}{dx} = V(x)\,!}$$